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OPTIMIZATION OF REINFORCEMENT
FOR A CLASS OF OPENINGS
IN PLATE STRUCTURES

by

S.K. Dhir

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COMPUTATION, MATHEMATICS & LOGISTICS DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

OPTIMIZATION OF REINFORCEMENT FOR A CLASS OF OPENINGS IN PLATE STRUCTURES

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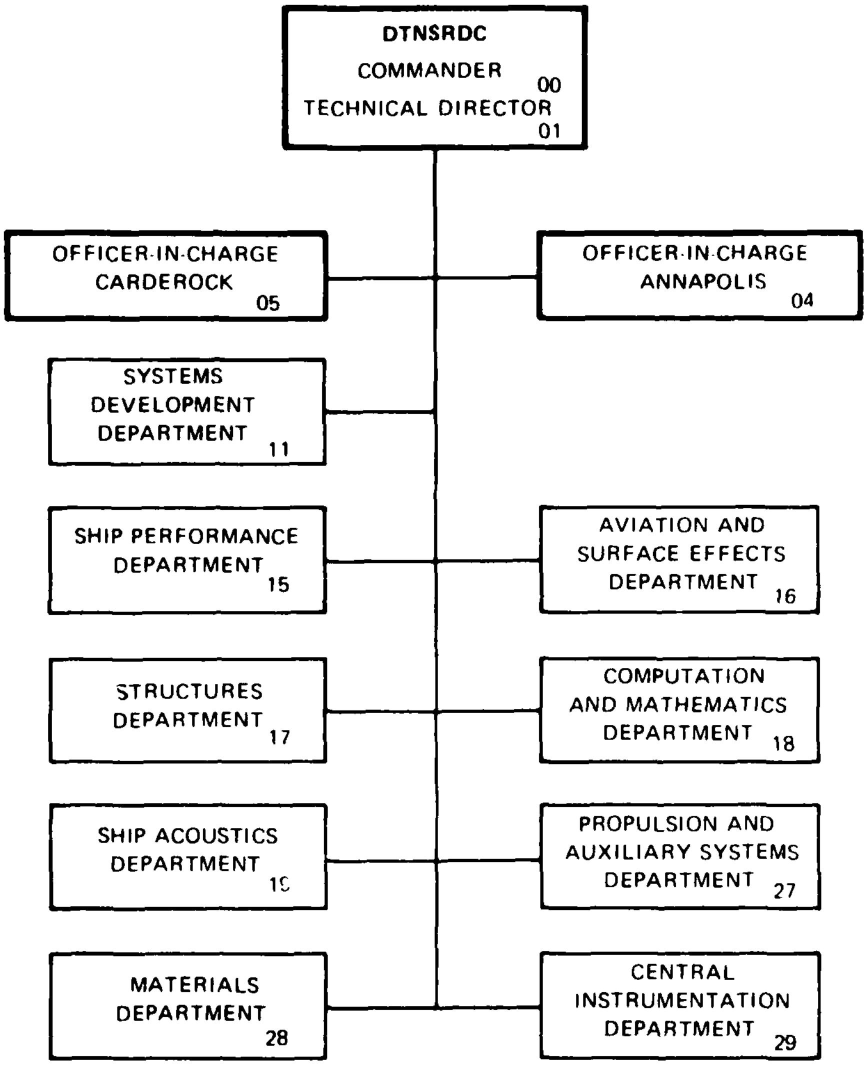

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A numerical/analytical procedure has been developed which yields the optimum amount of reinforcement for a given hole shape in a large elastic plate under prescribed boundary stresses. This procedure is based on determining the usual two complex potentials which describe the entire stress field, constructing the strain energy density function in terms of the unknown amount of reinforcement, integrating this function around the opening boundary, and finally minimizing this integral with respect to the reinforcement. → next page

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ABSTRACT

A numerical/analytical procedure has been developed which yields the optimum amount of reinforcement for a given hole shape in a large elastic plate under prescribed boundary stresses. This procedure is based on determining the usual two complex potentials which describe the entire stress field, constructing the strain energy density function in terms of the unknown amount of reinforcement, integrating this function around the opening boundary, and finally minimizing this integral with respect to the reinforcement.

The method is first developed for a general hole shape and then demonstrated in some detail for a circular and a square-like opening.

INTRODUCTION

The use of reinforcements at the boundaries of openings is standard practice in ship and aircraft construction. The amount of reinforcement to be used is determined, in most cases, by somewhat arbitrary decisions regarding the percentage reduction it produces in the boundary stress maximums. Although intrinsically there is nothing wrong with this approach, it seems to lack a rational basis or criterion for helping the structural designer select a specific amount of reinforcement. The procedure presented in this paper is an effort to provide such a basis. Rather than determining the stress field corresponding to a given geometry of the opening and of the reinforcement, this procedure seeks to determine that reinforcement which minimizes a certain meaningful integral related to the boundary strain energy. In this way it is an inverse elasticity problem. Some investigators[1-3], in studying the related problem of optimizing unreinforced notch shapes in plates, have concluded that uniform tangential stress at the notch boundary would, in general, lead to the smallest stress concentrations. Intuitively, it appears justified to assume that in the case of reinforced notch boundaries the requirement of uniformity of boundary stresses and/or stress related quantities, such as strain energy etc., at the notch boundary would lead to more desirable designs. The optimization rationale used in the present procedure is based on this argument.

In general, stress analysis of a non-circular opening reinforced with a thin member of uniform cross section is very difficult because it requires the satisfaction of a boundary condition which contains an irrational term. However, the use of MACSYMA (a symbolic manipulation language developed at MIT and in use at DTNSRDC) makes it possible to solve such problems, since a larger number of terms can be retained and manipulated in various expansions without losing track of them in the enormously long and complex algebraic expressions.

The general form of the boundary condition used in this method is developed in reference 4. The special cases of reinforcement of a circular opening and a square-like opening are discussed in greater detail and actual numerical results are included.

This paper is a logical extension of the work described in reference 5 which dealt with the optimization of the shape of a class of unreinforced openings in large plates.

MATHEMATICAL PRELIMINARIES

An opening of general shape in a large elastic plate of isotropic material of unit thickness is reinforced by a thin member of cross-sectional area A capable of withstanding axial forces only. Then, if the opening can be mapped into a unit circle by a function $z(\zeta)$, the equivalence of complex forces between those in the reinforcement and those in the plate at the opening boundary ($\zeta = \sigma$) is given by [4]

$$\phi(\sigma) + \frac{z(\sigma)}{\overline{z'(\sigma)}}\,\overline{\phi'(\sigma)} + \overline{\psi(\sigma)} = \sigma P \sqrt{\frac{z'(\sigma)}{\overline{z'(\sigma)}}} + C \qquad (1)$$

where $\phi(\sigma)$ and $\psi(\sigma)$ are the values of the functions $\phi(\zeta)$ and $\psi(\zeta)$ at the opening boundary, P is the axial force in the reinforcement, σ equals $e^{i\beta}$, and C is an arbitrary constant. Equivalence of tangential strain at the boundary requires

$$P = A\,(\sigma_\beta - \nu\sigma_\alpha) \qquad (2)$$

and the elastic equilibrium of a boundary element requires [4]

$$\sigma_{\alpha} = P \sqrt{z'\bar{z}'}$$

$$\tau_{\alpha\beta} = - \frac{1}{\sqrt{z'\bar{z}'}} \frac{\partial P}{\partial \beta} \tag{3}$$

where the argument σ of the function z has been omitted for brevity and prime indicates differentiation with respect to σ .

The well-known relations between the stresses, σ_{α} , σ_{β} , and $\tau_{\alpha\beta}$ and the functions, $\phi(\zeta)$ and $\psi(\zeta)$, and Equations (2) and (3) can be used to show that

$$P \sqrt{z'\bar{z}'} \left[A (1+\nu) + \sqrt{z'\bar{z}'} \right] = 2A (\phi' \bar{z}' + \bar{\phi}' z') \tag{4}$$

Equation (1) can be modified to

$$(\phi + \bar{\psi}) \bar{z}' + z \bar{\phi}' = \sigma P \sqrt{z'\bar{z}'} \tag{5}$$

where the argument σ has been omitted, for brevity, for the ϕ , ψ , and z functions; prime indicates differentiation with respect to the argument σ ; and a bar (-) represents the complex conjugate of the function. Since P is a real quantity, it is possible to represent

$$P \sqrt{z'\bar{z}'} = \sum_{n=0} c_n (\sigma^n + \frac{1}{\sigma^n}) \tag{6}$$

where the c_n are real. Functions $\phi(\zeta)$ and $\psi(\zeta)$ are known to have the following form:

$$\phi(\zeta) = S\zeta + \sum \frac{a_n}{\zeta^n}$$

$$\psi(\zeta) = D\zeta + \sum \frac{b_n}{\zeta^n} \tag{7}$$

where

$$S = \frac{p + q}{4}$$

$$D = -\frac{p - q}{2} e^{-2i\theta}$$

and p and q are uniform stresses at infinity at an angle θ to the x- and y-axes, respectively. The mapping function $z(\zeta)$ can be conveniently represented by

$$z(\zeta) = R\left(\zeta + \sum \frac{m_n}{\zeta^n}\right) \tag{8}$$

where m_n are known constants and R is a size factor which will be assumed unity.

The problem can now be stated in two parts: (a) For a given mapping function $z(\zeta)$, determine the functions $\phi(\zeta)$ and $\psi(\zeta)$ as a function of the area of reinforcement A; and (b) determine that value of A which minimizes a certain meaningful integral related to the boundary strain energy.

Clearly the solution of part (a), which forms the basis for the solution of part (b), would require the determination of a_n and b_n such that Equations (4) and (5) are satisfied.

Substitution of Equations (6), (7) and (8) into the boundary condition, Equation (5), and its ensuing solution generates one set of linear simultaneous Equations between a_n and c_n and another set between a_n, b_n, and c_n, both of which satisfy this boundary condition. Values of a_n in terms of c_n can be determined from the first set and substituted in Equation (4) to determine c_n in terms of A by using an expansion of the irrational term $\sqrt{z'\bar{z}'}$. Taylor series expansion of this term at $\zeta = 0$ appeared to be a reasonable choice, and it was seen that, for the demonstration problem of a square-like hole, an accuracy of 1 in 10^6 could be obtained at the hole boundary by using the first eight terms after the constant term. Once the c_n are known, the two sets of simultaneous equations can, in principle, be solved to obtain a_n and b_n in terms of the unknown quantity A. The

basic information for the optimization problem is then available.

OPTIMIZATION

The strain energy density, V_o, of a plane elastic system is given by

$$V_o = \frac{1}{2E}\left[\sigma_\alpha^{\;2} + \sigma_\beta^{\;2} - 2\nu\sigma_\alpha\sigma_\beta + 2(1+\nu)\tau_{\alpha\beta}^{\;\;2}\right] \tag{8}$$

With Equations (2) and (3), this expression for V_o can be transformed to

$$V_o = \frac{1}{4E}\left\{\left[\frac{2}{A^2} + \frac{2(1+\nu)}{z'\bar{z}'}\right] P^2 + \frac{4(1+\nu)}{z'\bar{z}'}\left(\frac{\partial P}{\partial \beta}\right)^2\right\} \tag{9}$$

The area of reinforcement, A, as it occurs in Equation (9) is a dimensionless quantity since it has been divided by the unit thickness of the plate as well as by the unit size factor R of the mapping function.

For specific loading conditions, the strain energy density, V_o, a function of β and A only, can be integrated with respect to β from 0 to 2π to obtain an integral, I, which can be interpreted as the strain energy in a thin region of variable thickness around the opening. This integral can now be minimized with respect to A to obtain that value of A which leads to the "most uniform" distribution of V_o around the opening. The term "most uniform" conveys the meaning that, of all the possible parametric variations of V_o corresponding to A, this particular distribution would have strongly attenuated peaks.[5]

A distinction must be made between this integral, I, and the actual strain energy, V, in a thin slice around the opening. The strain energy itself will be obtained by integrating V_o with respect to ds. Since ds is given by $\sqrt{z'\bar{z}'}\, d\beta$, the strain energy will be

$$V = \int V_o \sqrt{z'\bar{z}'}\, d\beta \tag{10}$$

It can be expected that a minimization of V with respect to A will lead to the "most uniform" distribution of $V_o \sqrt{z'\bar{z}'}$ rather than that of V_o around the opening. The integral, I, was previously calculated, within a constant, in reference (5) for a square-like unreinforced opening to determine its optimum shape. For S = .25 and D = -.5 the integral, I, was found to have a minimum value of 0.3627 at m = -.05. This value of m produced a very desirable stress distribution and the lowest stress concentrations since for an unreinforced opening the integrand, V_o, of I was σ_β^2. For comparison, when this case was later repeated by minimizing V such that the integrand was $\sigma_\beta^2 \sqrt{z'\bar{z}'}$, the minimum value of V within the same constant was found to be 0.3705 at m = -.07. The corresponding stress distribution around the opening is less desirable than when m = -.05. In fact, the highest stress concentration for m = -.07 was 2.505 versus 2.472 for m = -.05. Nevertheless, the $\sigma_\beta^2 \sqrt{z'\bar{z}'}$ distribution can be expected to be "most uniform" at m = -.07 and the σ_β^2 - distribution is the "most uniform" at m = -.05. From an engineering stand point the "most uniform" σ_β^2 - distribution will obviously be preferable. Other more mathematical reasons for choosing to minimize the integral of σ_β^2 are included in reference 5.

The preceding discussion justifies the use of the integral, I, rather than the strain energy, V, as the quantity to be minimized for determining the optimum A. In summary, minimization of I should, in general, be expected to lead to the "most uniform" V_o - distribution and minimization of V to the "most uniform" $V_o \sqrt{z'\bar{z}'}$ - distribution. It can also be expected, as evidenced in the case of the unreinforced square opening, that "most uniform" V_o - distribution would result in smaller V_o values.

At this point it should be mentioned that consideration was also given to determining optimum A based on attenuation of other relevant quantities such as the distortion energy density (equivalent to attenuation of the octahedral stress), the dilatation energy density, the maximum shear stress, and the maximum principal stress. All these quantities except the last are either as difficult or, in fact, simpler to attenuate than the strain energy density. Since the attenuation of the maximum principal stress is in general, considerably more difficult to perform (and not necessarily most desirable), the strain energy density was selected, in the case of the square-like hole, as a

typical quantity to be attenuated. In the case of the circular hole other quantities were also investigated for comparison.

EXAMPLES

1. Circular Opening

In this case the irrational terms in Equations (3), (4), and (5) disappear, so the manipulations are rather straight forward. Another point to be noted is that for a circular opening the strain energy, V, is the same as the integral I. Omitting all the intermediate manipulations, the strain energy, V, can be given by

$$V = I = \frac{8\pi}{E}\left[\frac{2(1 + 0.91A^2)S^2}{(1 + 1.3A)^2} + \frac{(1 + 11.31A^2)D^2}{(1 + 3.3A)^2}\right] \tag{11a}$$

the energy of distortion, V_D, or the integral of the square of octahedral stress, τ^2_{oct}, can be given by

$$V_D = \int_0^{2\pi} \frac{3.9\tau^2_{oct}}{2E}\, d\beta = \frac{13.867\pi}{2E}\left[\frac{2(1-0.4A+0.79A^2)S^2}{(1 + 1.3A)^2} + \frac{(1-0.4A+12.79A^2)D^2}{(1 + 3.3A)^2}\right] \tag{11b}$$

the energy due to change in volume, V_v, can be given by

$$V_v = \frac{2.133\pi}{2E}\left[2S^2 + \frac{(1 + 1.3A)^2}{(1 + 3.3A)^2}D^2\right] \tag{11c}$$

and finally the integral, T, of the square of maximum shear stress can be given by

$$T = 16\pi\left[\frac{2(1 - 1.4A + 0.49A^2)S^2}{(1 + 1.3A)^2} + \frac{(1 - 1.4A + 16.49A^2)D^2}{(1 + 3.3A)^2}\right] \tag{11d}$$

TABLE 1 - OPTIMUM VALUES OF REINFORCEMENT FOR A CIRCULAR HOLE SUBJECT TO VARIOUS TYPES OF LOADS:

ATTENUATED QUANTITY / LOAD CASES	STRAIN ENERGY DENSITY		DISTORTION ENERGY DENSITY		VOLUME ENERGY DENSITY		MAX SHEAR STRESS SQUARED	
	V_o	A	V_D	A	V_v	A	T	A
Isotropic S=0.5; D=0	$\frac{2.8\pi}{2E}$	1.4286	$\frac{1.733\pi}{2E}$	1.4286	$\frac{1.067\pi}{2E}$	-*	0	1.4286
Uniaxial S=0.25; D=-0.5	$\frac{3.074\pi}{2E}$	.4429	$\frac{2.584\pi}{2E}$	.4033	$\frac{0.349\pi}{2E}$	∞**	2.717π	.3422
Pure Shear S=0; D=-1	$\frac{8.15\pi}{2E}$	.2918	$\frac{7.072\pi}{2E}$	.2602	$\frac{0.331\pi}{2E}$	∞**	8π	.2127

* - Indicates that V_v is independent of A

** - Infinity refers to a rigid reinforcement

The quantities V, V_D, V_v, and T were minimized to determine the optimum values of A. These computations were performed for a number of load cases as shown in Table 1. The values of A at which the minimums occurred and the corresponding minimized quantities are included in this table.

The fact that such optimum reinforcements can exist is intriguing. A popular definition of A is the ratio of area replaced as reinforcement to the area removed by the opening. Thus in the uniaxial case, if approximately 44.3% of the area removed by the opening is replaced by the reinforcement, the resulting stress distribution would be very desirable, i.e., one that would result in a relatively smooth boundary strain energy density distribution. The actual stress distributions corresponding to the cases in Table 1 can be obtained using equations derived in reference 6.

II. Square Opening

The algebraic manipulations in this case are considerably more involved and lengthier than for the circular opening. A square opening described by

$$z = \sigma - \frac{0.05}{\sigma^3} \qquad (12)$$

was chosen for this example because of its optimum square-like shape[5]. Solution of Equations (4) and (5) and integration of Equation (9) required the use of the irrational terms $(z'\bar{z}')^{\frac{1}{2}}$, $(z'\bar{z}')^{-\frac{1}{2}}$, and $(z'\bar{z}')^{-1}$ which were conveniently represented by Taylor series of the type $\sum_{n=0}^{r} G_{4n}(\sigma^{4n} + \sigma^{-4n})$, c.f. Equation (15). In all cases truncation after eight terms could maintain an accuracy of 1 in 10^6 in the boundary values of these expansions. Equation (5) produced two sets of equations, the solution of one of which is given by

$$\left.\begin{aligned} a_{2n-1} - \delta_{n,1} m a_1 &= c_{2n} + 3ma_{2n+3} + \delta_{n,2} mS - \delta_{n,1} D \\ a_{2n} &= 0 \end{aligned}\right\} ; \; n \geq 1 \qquad (13)$$

where m = -0.05. It is easy to see from Equation (13) that, if either a_n or c_n is truncated, the values of individual a_n can be easily determined in terms of c_n. Because of the small value of m, the final results converged within 1 in 10^4 when the a_n beyond n = 13 were truncated. Once the a_n were determined, the b_n could be determined using the other set of equations. However, this procedure was not necessary since the alternative procedure for accomplishing the solution, described in the next paragraph, was more direct.

The a_n and Taylor expansion of $\sqrt{z'\bar{z}'}$ were substituted in Equation (4) to obtain c_n, given by

$$\left.\begin{aligned} c_{2n-1} &= 0 \quad ; \; n \geq 1 \\ c_{4n} &= \left(\sum_{k=1}^{5} p_{4n,k} A^k\right) S \Big/ \sum_{k=0}^{5} q_k A^k \quad ; \; 4 \geq n \geq 0 \\ c_{4n-2} &= \left(\sum_{k=1}^{4} p_{(4n-2),k} A^k\right) D \Big/ \sum_{k=0}^{4} r_k A^k \quad ; \; 4 \geq n \geq 1 \end{aligned}\right\} \qquad (14)$$

where p_n , q_n , and r_n are known coefficients but are not included here.

Once the c_n were obtained, the problem was basically solved as a function of the area of reinforcement A. The c_n were substituted in Equation (6) to obtain $P\sqrt{z'\bar{z}'}$ which, in turn, was substituted in Equation (9) to obtain V_o . The stresses σ_α , σ_β , and $\tau_{\alpha\beta}$ could be obtained using Equations (2) and (3). In order to integrate V_o , the following Taylor expansions were used:

$$\left.\begin{aligned} (z'\bar{z}')^{-\frac{1}{2}} &= \sum_{n=0}^{8} L_{4n}(\sigma^{4n} + \sigma^{-4n}) \\ (z'\bar{z}')^{-1} &= \sum_{n=0}^{8} M_{4n}(\sigma^{4n} + \sigma^{-4n}) \end{aligned}\right\} \tag{15}$$

These expansions, with the now known $P\sqrt{z'\bar{z}'}$, yielded

$$\left.\begin{aligned} P &= \sum_{n=0}^{8} P_{2n}\cos 2n\beta \\ P(z'\bar{z}')^{-\frac{1}{2}} &= \sum_{n=0}^{8} Q_{2n}\cos 2n\beta \end{aligned}\right\} \tag{16}$$

which were used to obtain the integral

$$I = \int_0^{2\pi} V_o \, d\beta = \frac{2I_1}{A^2} + 2(1-\nu^2)I_2 + 4(1+\nu)I_3 \tag{17}$$

where

$$\left.\begin{aligned} I_1 &= \frac{\pi}{2E}\left(P_o^{\,2} + \frac{1}{2}\sum_{n=1}^{8} P_{2n}^{\,2}\right) \\ I_2 &= \frac{\pi}{2E}\left(Q_o^{\,2} + \frac{1}{2}\sum_{n=1}^{8} Q_{2n}^{\,2}\right) \\ I_3 &= \frac{\pi}{2E}\, M_o\left(\sum_{n=1}^{8} (2nP_{2n})^2 + \sum_{n=1}^{8} 2n^2 M_{4n} P_{2n}^{\,2} + \sum_{n=1}^{8} M_{4n}R_{4n}\right) \end{aligned}\right\} \tag{18}$$

The R_n , given as sums of products $P_m P_n$, are coefficients of cos $n\beta$ in the expression $\left(\frac{\partial P}{\partial \beta}\right)^2$. The minimum value of I, which in Equation (17) is a function of A, S, and D only, was determined by simply evaluating I at

various values of A for a selected set of values of S and D. In fact, the three cases considered were the isotropic case (S = .5, D = 0), the uniaxial case (S = .25, D = -.5), and the pure shear case (S = 0, D = -1). The corresponding A's at which the minimum values of I occurred were 0.90, 0.41, and 0.29. These minimum values of I could be directly compared to those of reference (5), if they were first multiplied by $\frac{2E}{16\pi}$. The I values for these three cases were found to be .2453, .1920, and .4771, respectively. It should be noted that since the value of m for all three cases was -0.05, the initially unreinforced shape is optimized only for the uniaxial case[5]. The minimization of I with respect to A for all three cases demonstrates that there is also an optimum amount of reinforcement for those openings which do not initially have an optimum shape. Table 2 gives the minimized values of I, within the constant, and the corresponding values of A for the uniaxial case only.

TABLE 2 - MINIMIZED VALUES OF I AT OPTIMUM VALUES OF A FOR A SQUARE OPENING

A	$\frac{2E}{16\pi} \cdot I$
0.000001	.36268
0.1	.24854
0.2	.20936
0.3	.19546
0.4	.19199
0.412	.19195
0.5	.19324
0.6	.19671
0.7	.20120
0.8	.20610
0.9	.21109
1.0	.21600

As expected, the value of $\frac{2E}{16\pi}$ I corresponding to A = .00001 ≈ 0 is the same as that of reference (5) for A = 0 (unreinforced). Table 2 also shows the variation of I with respect to A and its minimum point at A = .412.

Figure 1 shows the stresses σ_α, σ_β, and $\tau_{\alpha\beta}$ for a reinforced square-like opening (m = -.05) with A = .412 and also, for comparison, the equivalent stress, σ_E, based on the strain energy theory of failure and the value of σ_β for the unreinforced opening subjected to uniaxial loading. The maximum value, 1.43, of σ_β in the reinforced case occurs approximately at $\beta = 80°$ and in the unreinforced case this maximum 2.47, occurs at approximately $\beta = 70°$.

Since the amount of area removed is not a constant by definition, in the case of a square opening with rounded corners, the percentage of area replaced can not be a constant either. For convenience, however, it can be referred to the half-width of the square. Thus the value of A should be divided by 1 + m, i.e., 0.95, and multiplied by 100 to obtain the reinforcement expressed as a percentage of the area removed.

CONCLUSIONS

1. It has been shown that for openings in plates an optimum amount of reinforcement exists which corresponds to a minimum value of a strain energy related integral.

2. It can be conjectured that the values of strain energy densities around a reinforced opening are relatively smooth when this strain energy related integral is a minimum.

3. The actual optimum values of reinforcement for a circular opening were found to be 142%, 44.3%, and 29.2% of the area removed by the opening for isotropic, uniaxial, and pure shear loadings, respectively.

4. The optimum values of reinforcement for a square-like opening were found to be 94.7%, 43.4%, and 30.5% of the area removed by the opening for isotropic, uniaxial, and pure shear loadings, respectively.

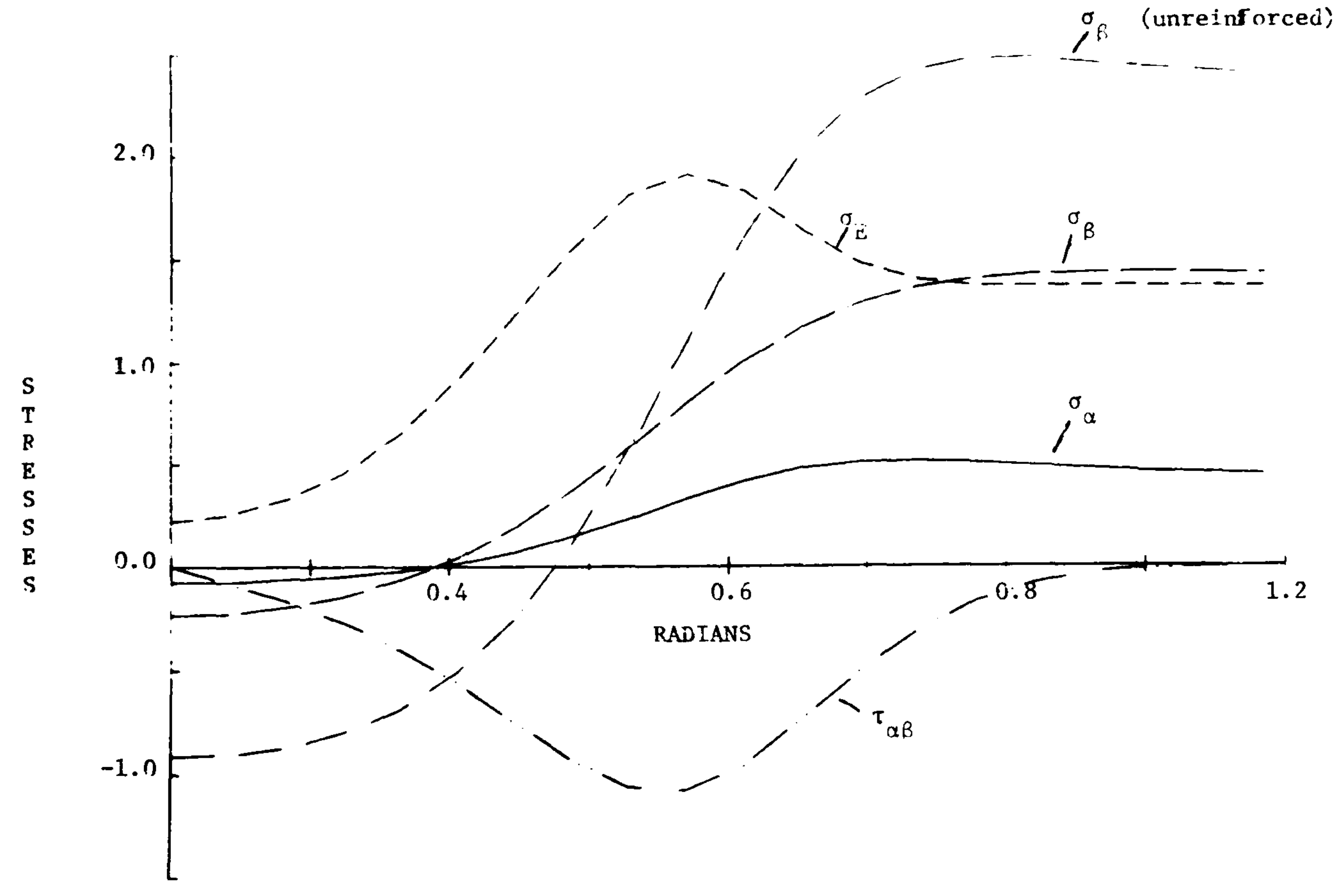

FIGURE 1 - Various Boundary Stresses Around a Square-Like Opening With Optimum Reinforcement

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